DÚ

$a^2 \cdot (x+1) - 2 \cdot (ax+2) = 0 \qquad a \in \mathbb{R}$ - parametr

1. $a^2x + a^2 - 2ax - 4 = 0$

$a^2x - 2ax + a^2 - 4 = 0 \qquad \Rightarrow$ LINEÁRNÍ

$x \cdot (a^2 - 2a) = 4 - a^2$

$a \cdot (a-2)$

$a = 0$

$0 - 4 = 0$

$-4 = 0$

$K = \{\}$

$a = 2$

$4(x+1) - 2(2x+2) = 0$

$4x + 4 - 4x - 4 = 0$

$0 = 0$

$K = \mathbb{R}$

$a \neq 0, \ a \neq 2$

$x = \frac{4 - a^2}{a(a-2)} = \frac{(2-a)(2+a)}{a(a-2)} = \frac{-a-2}{a}$

Závěr:

$a = 0 \ \ldots \ K = \{\}$

$a = 2 \ \ldots \ K = \mathbb{R}$

$a \in \mathbb{R} - \{0; 2\} \ \ldots \ K = \left\{\frac{-a-2}{a}\right\}$

2. $x^2 - \lambda x + 1 - 2\lambda^2 = 0 \qquad \lambda \in \mathbb{R}$ - parametr

$D = (-\lambda)^2 - 4 \cdot 1 \cdot (1 - 2\lambda^2) = \lambda^2 - 4 + 8\lambda^2 = 9\lambda^2 - 4$

$D = 0$

$9\lambda^2 - 4 = 0$

$(3\lambda - 2)(3\lambda + 2) = 0$

$\lambda_1 = \frac{2}{3} \ \ldots \ x_{1,2} = \frac{\lambda}{2} = \frac{\frac{2}{3}}{2} = \frac{2}{6} = \frac{1}{3}$

$\lambda_2 = \frac{-2}{3} \ \ldots \ x_{1,2} = \frac{\lambda}{2} = \frac{-\frac{2}{3}}{2} = -\frac{1}{3}$

$D > 0$

$9\lambda^2 - 4 > 0$

$(3\lambda - 2)(3\lambda + 2) > 0$

(+) $-\frac{2}{3}$ (−) $\frac{2}{3}$ (+)

$\lambda \in (-\infty; -\frac{2}{3}) \cup (\frac{2}{3}; \infty)$

$x_{1,2} = \frac{\lambda \pm \sqrt{9\lambda^2 - 4}}{2}$

$D < 0$

$9\lambda^2 - 4 < 0$

$\lambda \in (-\frac{2}{3}; \frac{2}{3})$

$x_{1,2} \in \{\}$

Závěr:

$\lambda = \frac{2}{3} \ \ldots \ K = \{\frac{1}{3}\}$

$\lambda = -\frac{2}{3} \ \ldots \ K = \{-\frac{1}{3}\}$

$\lambda \in (-\infty; -\frac{2}{3}) \cup (\frac{2}{3}; \infty) \ \ldots \ K = \left\{\frac{\lambda \pm \sqrt{9\lambda^2 - 4}}{2}\right\}$

$\lambda \in (-\frac{2}{3}; \frac{2}{3}) \ \ldots \ K = \{\}$

3. $\frac{m}{x} - \frac{4}{mx} = 1 - \frac{2}{m}$ $\quad m \in \mathbb{R}$ - parametr

$\downarrow$

$m = 0$ ... ROVNICE NEMÁ SMYSL

<u>$m \neq 0$:</u> podmínka $x \neq 0$

rovnici vynásobíme mx

$m^2 - 4 = mx - 2x$

$x(m-2) = m^2 - 4$

$m = 2$

$\frac{2}{x} - \frac{4}{2x} = 1 - \frac{2}{2}$

$\frac{2}{x} - \frac{2}{x} = 0 \quad x \neq 0$

$0 = 0$ $\quad \forall$

$K = \mathbb{R} - \{0\}$

$m \neq 2$:

$x = \frac{m^2-4}{m-2} = m + 2$

KONTROLA PODMÍNKY : $m + 2 = 0$

$m = -2$

$\downarrow$

PRO TOTO m JE PODMÍNKA PORUŠENÁ

$\Rightarrow K = \{\}$

Závěr: $m = 0$ ... rovnice nemá smysl

$m = 2$ .... $K = \mathbb{R} - \{0\}$

$m = -2$ .... $K = \{\}$

$m \in \mathbb{R} - \{0, 2, -2\}$ ... $K = \{m+2\}$

4. $2mx^2 + mx + 1 = 0$

$m = 0$

$1 = 0$

$K = \{\}$

$m \neq 0$

$D = m^2 - 4 \cdot 2m \cdot 1 = m^2 - 8m$

Závěr:

$m \in \langle 0, 8) \ldots K = \{\}$

$m = 8 \ldots K = \{-\frac{1}{4}\}$

$m \in (-\infty, 0) \cup (8, \infty) \ldots$

$\ldots K = \left\{\frac{-m \pm \sqrt{m^2 - 8m}}{4m}\right\}$

$D = 0$

$m^2 - 8m = 0$

$m(m-8) = 0$

$m_1 = 0$ .... nelze !

$m_2 = 8 \ldots x_{1,2} = \frac{-m}{4m} = \frac{-1}{4}$

$D > 0$

$m^2 - 8m > 0$

$m(m-8) > 0$

$\oplus \quad - \quad \oplus$

$0 \qquad 8$

$m \in (-\infty, 0) \cup (8, \infty)$

$x_{1,2} = \frac{-m \pm \sqrt{m^2 - 8m}}{4m}$

$D < 0$

$m \in (0, 8)$

$K = \{\}$

Dú

(5.) $3x + ay = 1$

$x + 2y = 3$

$x = 3 - 2y$

$3(3-2y) + ay = 1$

$9 - 6y + ay = 1$

$ay - 6y = -8$

$y(a-6) = -8$

$a = 6$

$3x + 6y = 1$

$x + 2y = 3 \mid \cdot 3$

$0 = -8$

$K = \{\}$

$a \neq 6$

$y = \frac{-8}{a-6}$

$x = 3 - 2 \cdot \frac{-8}{a-6} = 3 + \frac{16}{a-6} = \frac{3a-18+16}{a-6} = \frac{3a-2}{a-6}$

$K = \left\{ \left[\frac{3a-2}{a-6} ; \frac{-8}{a-6} \right] \right\}$

Závěr: $a = 6 \dots K = \{\}$

$a \in R - \{6\} \dots K = \left\{ \left[\frac{3a-2}{a-6} ; \frac{-8}{a-6} \right] \right\}$

(6.) $p \cdot (p-1)\, x < 1$ $\quad p$ - parametr

$p = 0$

$0 < 1$

$x \in R$

$p = 1$

$0 < 1$

$x \in R$

$p \cdot (p-1) > 0$

$\oplus \quad 0 \quad - \quad 1 \quad \oplus$

$p \in (-\infty, 0) \cup (1, \infty)$

$x < \frac{1}{p(p-1)}$

$x \in \left(-\infty, \frac{1}{p(p-1)}\right)$

$p(p-1) < 0$

$+ \quad 0 \quad - \quad 1 \quad +$

$p \in (0, 1)$

$x > \frac{1}{p(p-1)}$

$x \in \left(\frac{1}{p(p-1)} ; \infty\right)$

Závěr: $p \in \{0, 1\} \dots K = R$

$p \in (-\infty, 0) \cup (1, \infty) \dots K = \left(-\infty, \frac{1}{p(p-1)}\right)$

$p \in (0, 1) \dots K = \left(\frac{1}{p(p-1)} ; \infty\right)$